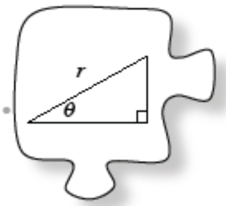
December 9, 2019

5.1.1 What if I know the hypotenuse?



Sine and Cosine Ratios

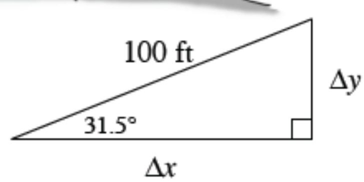
In the previous chapter, you used the idea of similarity in right triangles to find a relationship between the acute angles and the lengths of the legs of a right triangle. However, we do not always work just with the legs of a right triangle—sometimes we only know the length of the hypotenuse. By the end of today's lesson, you will be able to use two new trigonometric ratios that involve the hypotenuse of right triangles.

5-1. THE STREETS OF SAN FRANCISCO

While traveling around the beautiful city of San Francisco, Juanisha climbed several steep streets. One of the steepest, Filbert Street, has a slope angle of 31.5° according to her guidebook.

Once Juanisha finished walking 100 feet up the hill, she decided to figure out how high she had climbed. Juanisha drew the diagram below to represent this situation.

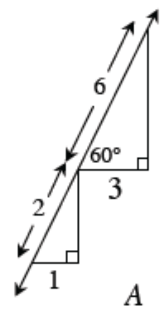
Can a tangent ratio be used to find Δy? Why or why not? Be prepared to share your thinking with the rest of the class.



Juanisha's Drawing

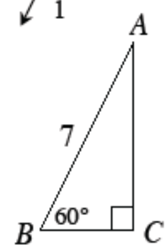
5-2. In order to find out how high Juanisha climbed in problem 5-1, you need to know more about the relationship between the ratios of the sides of a right triangle and the slope angle.

a. Use **two different strategies** to find Δy for the slope triangles shown in the diagram at right.



b. Find the ratio $\frac{\Delta x}{\text{hypotenuse}}$ for each triangle. Why must these ratios be equal?

c. Find BC and AC in the triangle at right. Show all work.



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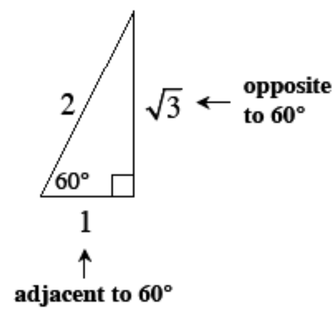
5-3. NEW TRIG RATIOS

In problem 5-2, you used a ratio that included the hypotenuse of $\triangle ABC$. There are several ratios that you might have used. One of those ratios is known as the **sine ratio** (pronounced "sign"). This is the ratio of the length of the side **opposite** the acute angle to the length of the **hypotenuse**.



For the triangle shown at right, the sine of 60° is $\frac{\sqrt{3}}{2} \approx 0.866$. This is written:

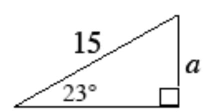
$$\sin 60° = \frac{\sqrt{3}}{2}$$

Another ratio comparing the length of the side **adjacent** to (which means "next to") the angle to the length of the **hypotenuse**, is called the **cosine ratio** (pronounced "co-sign"). For the triangle above, the cosine of 60° is $\frac{1}{2} = 0.5$. This is written:

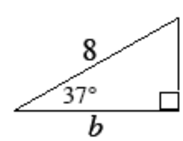
$$\cos 60° = \frac{1}{2}$$

a. Like the tangent, your calculator can give you both the sine and cosine ratios for any angle. Locate the "sin" and "cos" buttons on your calculator and use them to find the sine and cosine of 60°. Does your calculator give you the correct ratios?

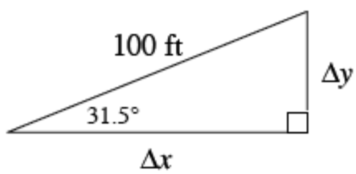
b. Use a trig ratio to write an equation and solve for a in the diagram at right. Does this require the sine ratio or the cosine ratio?



c. Likewise, write an equation and solve for b for the triangle at right.

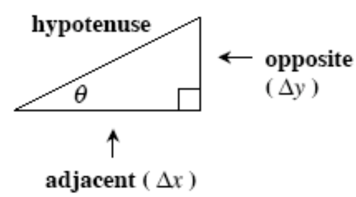


5-4. Return to the diagram from Juanisha's climb in problem 5-1. Juanisha still wants to know how many feet she climbed vertically when she walked up Filbert Street. Use one of your new trig ratios to find how high she climbed.

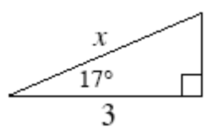


Juanisha's Drawing

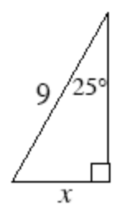
5-5. For each triangle below, decide which side is <u>opposite</u> and which is <u>adjacent</u> to the given acute angle. Then determine which of the three trig ratios will help you find x. Write and solve an equation.



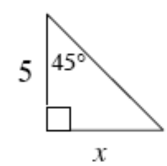
a.



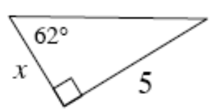
b.



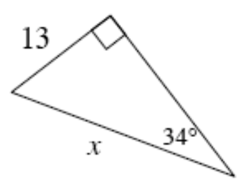
c.



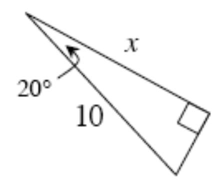
d.



e.



f.



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5-6. TRIANGLE TOOLKIT

Obtain a Lesson 5.1.1 Resource Page ("Triangle Toolkit") from your teacher. This will be a continuation of the Geometry Toolkit you started in Chapter 1. Think about the tools you have developed so far to solve for the measure of sides and angles of a triangle. Then, in the space provided, add a diagram and a description of each tool you know. In later lessons, you will continue to add new triangle tools to this toolkit, so be sure to keep this resource page in a safe place. At this point, your toolkit should include:

- Pythagorean Theorem
- Tangent
- Sine
- Cosine

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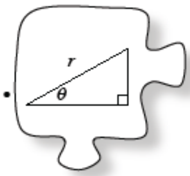
5.1.2 Which tool should I use?



Selecting a Trig Tool

You now have several tools that will help you find the length of a side of a right triangle when given any acute angle and any side length. But how do you know which tool to use? And how can you identify the relationships between the sides and the given angle?

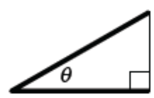
Today you will work with your team to develop **strategies** that will help you identify if cosine, sine, or tangent can be used to solve for a side of a right triangle. As you work, be sure to share any shortcuts you find that can help others identify which tool to use. As you work, keep the focus questions below in mind.

Is this triangle familiar? Is there something special about this triangle?

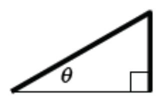
Which side is opposite the given angle? Which is adjacent?

Which tool should I use?

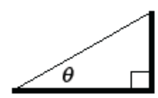
5-12. Obtain the Lesson 5.1.2 Resource Page from your teacher. On it, find the triangles shown below. Note: the diagrams are not drawn to scale.



$\cos\theta = \frac{adj}{hyp}$



$\sin\theta = \frac{opp}{hyp}$

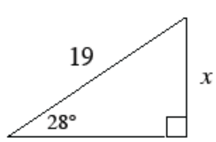


$\tan\theta = \frac{opp}{adj}$

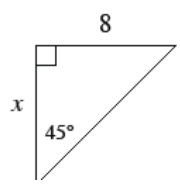
With your study team:

- Look through all the triangles first and see if any look familiar or are ones that you know how to answer right away without using a trig tool.
- Then, for all the other triangles, identify which tool you should use based on where the reference angle (the given acute angle) is located and which side lengths are involved.
- Write and solve an equation to find the missing side length.

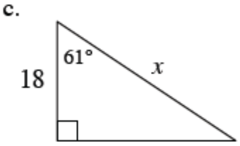
a.



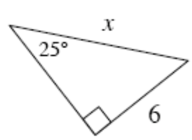
b.



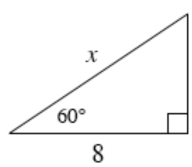
c.



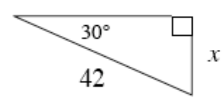
d.



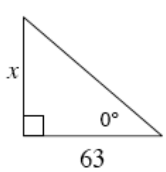
e.



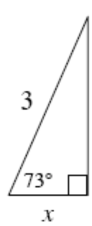
f.



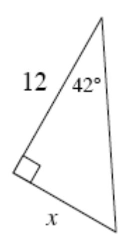
g.



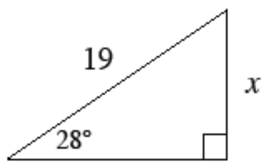
h.



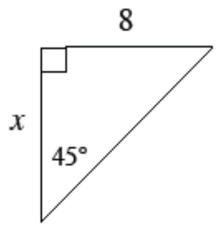
i.



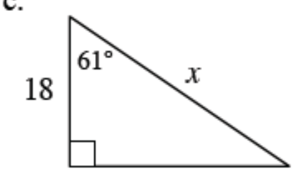
5-12. a.



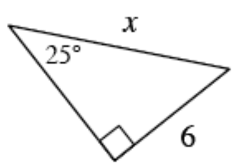
b.



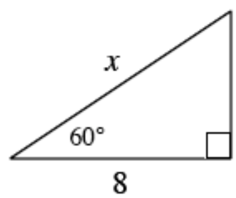
c.



d.



e.



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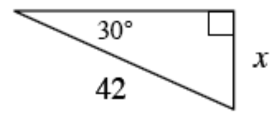
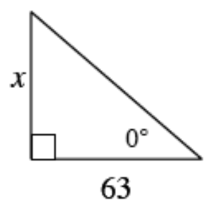
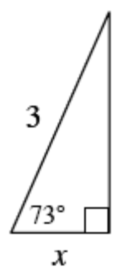


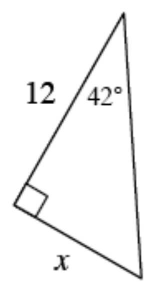
g.



h.



i.



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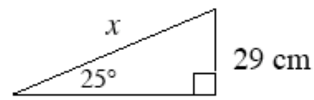
5-13. Marta arrived for her geometry test only to find that she forgot her calculator. She decided to complete as much of each problem as possible.

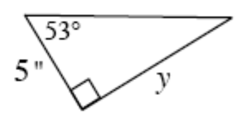
a. In the first problem on the test, Marta was asked to find the length x in the triangle shown at right. Using her algebra skills, she wrote and solved an equation. Her work is shown below. Explain what she did in each step.



$$\sin 25° = \frac{29}{x}$$
$$x(\sin 25°) = 29$$
$$x = \frac{29}{\sin 25°}$$

b. Marta's answer in part (a) is called an **exact answer**. Now use your calculator to help Marta find the **approximate** length of x.

c. In the next problem, Marta was asked to find y in the triangle shown at right. Find an exact answer for y without using a calculator. Then use a calculator to find an approximate value for y.



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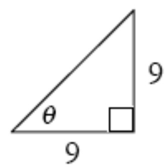
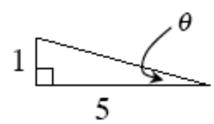
5-14. In problem 5-12, you used trig tools to find a side length. But do you have a way to find an angle? **Examine** the triangles below. Do any of them look familiar? How can you use information about the side lengths to help you figure out the reference angle (θ)? Your Trig Table from Chapter 4 may be useful.

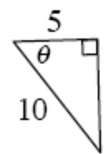
a.



b.



c.



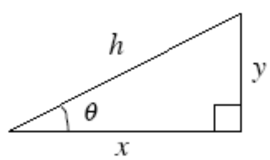
5-15. Write a Learning Log entry explaining how you know which trig tool to use. Be sure to include examples with diagrams and anything else that would be useful to refer to later. Title this entry, "Choosing a Trig Tool" and include today's date.

METHODS AND MEANINGS

MATH NOTES

Trigonometric Ratios

You now have three **trigonometric ratios** you can use to solve for the missing side lengths and angle measurements in any right triangle. In the triangle below, when the sides are described relative to the angle θ, the opposite leg is y and the adjacent leg is x. The hypotenuse is h regardless of which acute angle is used.



$$\tan\theta = \frac{\textit{opposite leg}}{\textit{adjacent leg}} = \frac{y}{x}$$

$$\sin\theta = \frac{\textit{opposite leg}}{\textit{hypotenuse}} = \frac{y}{h}$$

$$\cos\theta = \frac{\textit{adjacent leg}}{\textit{hypotenuse}} = \frac{x}{h}$$

In some cases, you may want to rotate the triangle so that it looks like a slope triangle in order to easily identify the reference angle θ, the opposite leg y, the adjacent leg x, and the hypotenuse h. Instead of rotating the triangle, some people identify the opposite leg as the leg that is always opposite (not touching) the angle. For example, in the diagram at right, y is the leg opposite angle θ.